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LETTER

Finite-Difference Beam-Propagation Method for Circularly Symmetric Fields

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SUMMARY Analysis of the propagation of circularly symmetric fields is made using the finite-difference beam-propagation method. After testing the accuracy of this method, we analyze the guided-mode transmission of connected fibers whose core radii are different. The propagation behavior of the unguided-mode field generated at the junction is revealed using a transparent boundary condition.

key words : *opto-electronics, electromagnetic theory*

1. Introduction

A method of analyzing a propagating field in cylindrical coordinates has received much attention, since the conventional beam-propagation method in Cartesian coordinates results in reduced computational efficiency for propagation problems in circularly symmetric waveguides. Feit and Fleck employed the fast Fourier transform to solve the Fresnel equation in cylindrical geometry(1). An alternative approach using the finite-difference techniques was also reported(2). In both methods, a Taylor series is used for the expansion of the exponential evolution operator. Although the order of the expansion can be determined according to the desired precision, the order is limited by the boundary condition(2). Consequently, it is inefficient to treat the problems where the appreciable field propagates towards the computational window edge.

In this Letter, we describe a simple method of analyzing the propagation of circularly symmetric fields. The formulation is based on the finite-difference techniques introduced in Cartesian coordinates(3)−(5). The present method has an advantage that a transparent boundary condition(6) is easily incorporated. Since there is no need to add extra zones for absorbing regions, the propagation problem including unguided modes can be treated efficiently. After testing the accuracy of the present method by the mode-mismatch loss evaluation, we analyze the guided-mode transmission of connected fibers whose core radii are different.

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2. Formulation

We consider the propagation of a circularly symmetric field of $E(r, z)$ in the optical fiber. The electric field is expressed as

$$E(r, z) = \mathrm{E}(r, z) \exp(-jkn_0 z) \qquad (1)$$

where n_0 is a reference refractive index, $k=2\pi/\lambda$ is the free space wavenumber, and λ is the vacuum wavelength.

The Fresnel equation for circularly symmetric fields takes the form

$$2jkn_0\frac{\partial \mathrm{E}}{\partial z} = \frac{\partial^2 \mathrm{E}}{\partial r^2} + \frac{1}{r}\frac{\partial \mathrm{E}}{\partial r} + k^2(n^2 - n_0^2)\mathrm{E} \qquad (2)$$

where $n=n(r, z)$ is the refractive index of the waveguide. Let $\mathrm{E}_i(z)$ be the electric field at $[(i-1)\Delta r, z]$ with $i=1, 2, \cdots, N$, and Δz be the propagation step length. Following the procedure used by Chung and Dagli(3), we obtain for $i>1$

$$\begin{aligned} &-a^+\mathrm{E}_{i-1}(z+\Delta z) + b^+\mathrm{E}_i(z+\Delta z) \\ &-a^-\mathrm{E}_{i+1}(z+\Delta z) \\ &\quad = a^+\mathrm{E}_{i-1}(z) + b^-\mathrm{E}_i(z) + a^-\mathrm{E}_{i+1}(z) \end{aligned} \qquad (3)$$

where $a^{\pm}=\Delta z\{0.5\mp 0.25/(i-1)\}/\Delta r^2$ and $b^{\pm}=\pm\Delta z/\Delta r^2 \mp 0.5\Delta zk^2(n^2-n_0^2)+2jkn_0$.

The indeterminate form at the origin $r=0$ can be evaluated by L'Hospital's rule(1). Taking into account the circular symmetry of the field, we obtain for $i=1$

$$c^+\mathrm{E}_1(z+\Delta z) - d\mathrm{E}_2(z+\Delta z) = c^-\mathrm{E}_1(z) + d\mathrm{E}_2(z) \qquad (4)$$

where $c^{\pm}=\pm 2\Delta z/\Delta r^2 \mp 0.5\Delta zk^2(n^2-n_0^2)+2jkn_0$ and $d=2\Delta z/\Delta r^2$.

As a result, propagating the beam at every step requires the solution of a tridiagonal set of linear equations, which can be done very efficiently. Furthermore, in contrast to the other methods(1),(2), a transparent boundary condition (TBC)(6) is easily incorporated in the present method. The TBC algorithm developed in the Cartesian coordinates can also be used in the cylindrical coordinates without any modification. The TBC algorithm leads to enhanced computational efficiency, particularly when the unguided-mode field is present.

3. Results

We analyze a step-index fiber whose core radius is $r_c = 5\ \mu$m. The refractive indices of the core and the cladding are $n_1 = 1.503$ and $n_0 = 1.5$, respectively, so that the normalized frequency is $V \cong 1.93$. The input field is the fundametal mode LP_{01}, which can be obtained analytically. A wavelength of $\lambda = 1.55\ \mu$m is used throughout this analysis.

As a test on the accuracy of the present method, we calculate the power loss of the eigenmode of the waveguide. The mismatch between input field and calculated output field is known to be a sensitive indicator of the accuracy of a beam propagation method[3].

Figure 1 shows the mode-mismatch loss as a function of propagation step length Δz for several values of Δr. Computational window dimension remains constant ($r_w = 50\ \mu$m), even when Δr is changed. That is, the number of transverse grid points is changed from 46 to 196, as Δr decreases from $r_c/4.5$ to $r_c/19.5$. The loss evaluation is made at a propagation distance of 1000 μm. Figure 1 indicates that the mode-mismatch loss is not a sensitive function of Δz. The loss decreases as Δr is decreased, since the approximation by finite differences is improved. We choose $\Delta r = r_c/19.5\ \mu$m, $\Delta z = 5\ \mu$m and $N = 196$ for the following analysis.

We now consider the guided-mode transmission problem shown in the inset of Fig. 2. The fibers identical to that used in Fig. 1 are connected through a fiber with a core radius of $1.92 r_c$. The other geometrical parameters of the interconnecting fiber are the same as those of the input and output fibers. Because the junctions are centered, the fundamental mode LP_{01} can only couple with the LP_{0m} modes.

The guided-mode transmission is defined as P_O/P_I, where P_I and P_O are the guided-mode power in the input and output fibers, respectively. Figure 2 shows that the guided-mode transmission oscillates as a function of the interconnecting fiber length L, owing to the effect of coherent coupling. As the L is increased, the oscillation becomes damped. This is due to the fact that the core radius of the interconnecting fiber is not large enough to support the LP_{02} mode, and thus the power coupled to the output fiber decreases as the L is increased. To clarify this fact, we decompose the propagating field into guided and unguided fields. Figure 3 shows the propagating field of unguided modes. It is clearly seen how the unguided-mode field is generated at the junction and is spread as it propa-

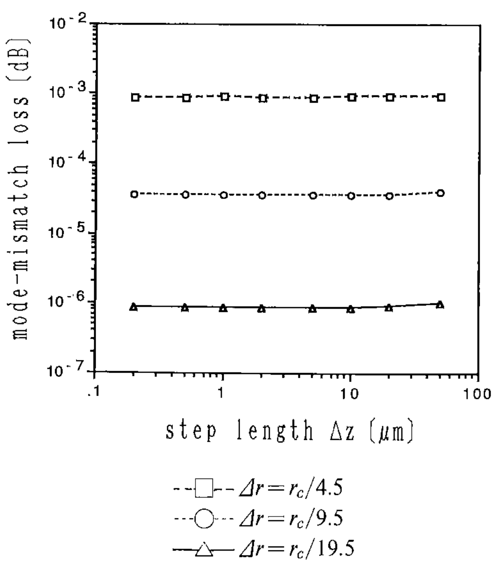

Fig. 1 Mode-mismatch loss as a function of propagation step length Δz.

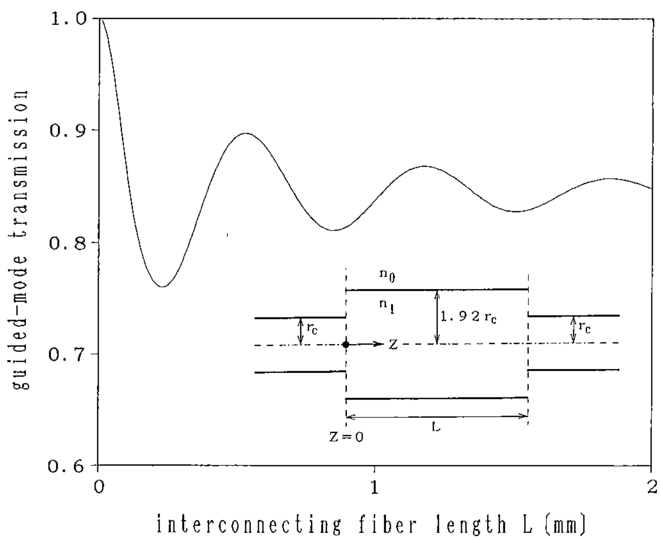

Fig. 2 Guided-mode transmission of coupled fibers as a function of interconnecting fiber length L.

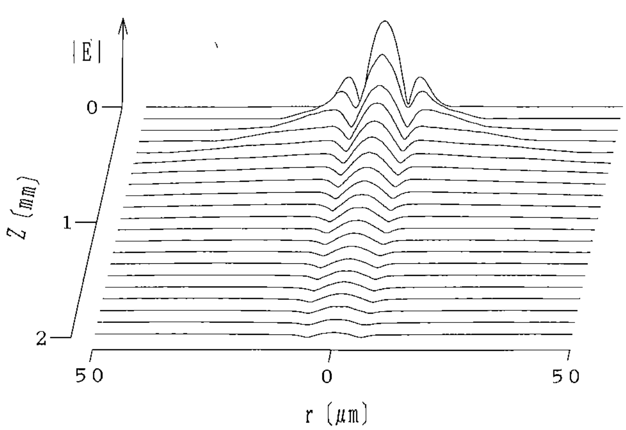

Fig. 3 Field propagation of unguided modes generated at the junction.

gates.

4. Conclusions

Analysis of the propagation of circularly symmetric fields has been made using the finite-difference beam-propagation method. The present method has an advantage that the transparent boundary condition is easily incorporated. The guided-mode transmission of connected fibers with different core radii is evaluated, and the behavior of the unguided-mode field generated at the junction is revealed.

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